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M.A. / M.Sc. (Third Semester)
EXAMINATION, Dec. - Jan., 2021-22
MATHEMATICS
Paper Third (C)
Mathematical Biology - I

[Time : Three Hours] *[Maximum Marks : 80]*

[Minimum Pass Marks : 16]

Note : Attempt all sections as directed

Section - A

(1 mark each)

(Objective / Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. The population growth model $\frac{dx}{dt} = rx, x(0) = x_0$ is called:
   (A) Verhulst model
   (B) Lotka-Voltera model
   (C) Prey-predator model
   (D) Malthus model

2. For the logistic growth model $\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$ choose the incorrect statement :
   (A) r is intrinsic growth rate
   (B) K is carrying capacity
   (C) $x_\infty = 0$ is stable and $x_\infty = K$ is unstable equilibrium
   (D) $x_\infty = 0$ is stable and $x_\infty = K$ is stable equilibrium

3. Choose the incorrect match for population growth model for intrinsic growth and the name who formulated it:
   (A) $r(x) = r \log \frac{K}{x}$, Gompertz
   (B) $r(x) = \frac{r(k - x)}{k + ax}$, F. Smith
   (C) $r(x) = r\left(1 - \left(\frac{x}{K}\right)^\theta\right)$, Hethcote
   (D) $r(x) = re^{\left(1 - \frac{x}{k}\right)} - d,$, Nisbet and Gurney

4. for the population grow model $x^1 = x(e^{3-x} - 1)$, choose the correct θ statement :
   (A) $x_\infty = 0$ is a stable equilibrium.
   (B) $x_\infty = 0$ is an unstable equilibrium.
   (C) $x_\infty = 3$ ia an unstable equilibrium.
   (D) None of these

5. Cobwebbing method is used to sovle graphically
   (A) Difference equations
   (B) Differential equations
   (C) Integral equations
   (D) Integro-differenctial equations
6. The difference equation $x_{n+1} = f(x_n)$, is linearized about equilibrium $x\infty$ by
   (A) Maclaurin's theorem
   (B) Taylaor's theorem
   (C) Lagrange's Theorem
   (D) Mean value theorm
7. For the difference equation $x_{n+1} = f(x_n)$, periodic points with period 2 are given by
   (A) f(f(n)) = 0
   (B) f(f(x)) = x
   (C) f(x) = x
   (D) f(f(f(x))) = 0
8. The model $\frac{dx}{dt} = x(\lambda - by)$, $\frac{dx}{dt} = y(-\mu - cy)$ is known as
   (A) Metered model
   (B) Ltka-Voltera model
   (C) Richer stock recruitment model
   (D) Bevernton and Holt stock recruitment model

9. Novick and Szilard (1950) and monod (1950) proposed models related to
   (A) chemostat
   (B) Fish
   (C) Dog
   (D) Human
10. In a systems of ardinary differential equations, If S is a periodic orbit, then index of S is
    (A) +1 (B) +2
    (C) +3 (D) +4
11. supose that $F_x(x, y) + Gy(x, y)$ is either strictly positive or strictly negative in a simply connected region D then there is no periodic oribit of $x^1 = F(x, y)$, $y^1 = G(x, y)$ in D this result is known as
    (A) Dulac theorem
    (B) Green theorem
    (C) Bendixson theorem
    (D) None of these
12. For the systems $x^1 = f(x, y)$, $y^1 = g(x, y)$, the curves $f(x, y) = 0$ and $g(x, y) = 0$ are called
    (A) limit cyle
    (B) periodic orbit
    (C) nullclines
    (D) None of these

13. A classic example of interacting population with oscillations observed by Hudson's Bay company in canada during the period 1821-1940 is related to

(A) mice and cat

(B) hare and lynx

(C) fox and lion

(D) none of these

14. choose the correct assumptions for kolmogorov model $x^1 = xf(x,y)$, $y^1 = yf(x,y)$

(A) $f_y(x,y) < 0$

(B) $g_x(x,y) > 0$

(C) $g_y(x,y) \le 0$

(D) None of these

15. Choose the incorrect statement:

(A) The situations in which the interaction of two species is beneficial is called mutualism.

(B) In facultative interaction, two species could survive seperately.

(C) In obligatory interaction, one species will become extinct without the assistance of the other.

(D) None of these

16. In the system $x^1 = x(1-x^2)$, -

(A) $x_\infty = 0$ is locally asympotically stable.

(B) $x_\infty = -1$ is locally asymptotically unstable

(C) $x_\infty = +1$ is locally asymptotically unstable

(D) None of above

17. If all eigen values of the community matrix of the system $x^1 = f(x),\ x \in |R^n$, at equilibrium ξ have negative real part, then the equilibrium is.

(A) unstable

(B) asymptotically stable

(C) global stable

(D) saddle

18. For discrete system $x_{n+1} = f(x_n)$, if at equilibrium x_∞, $|f^1(x_\infty)| < 1$ then the equilibrium x_∞

(A) unstable

(B) saddle

(C) asymptotically stable

(D) global stable

19. The difference equation $x_{n+1} = x_n e^{3-x_n}$, has equilibrium

(A) $x_\infty = 0$ as asymptotically stable.

(B) $x_\infty = 3$ as asymptotically stable.

(C) $x_\infty = 0$ asymptotically stable and $x_\infty = 3$ unstable.

(D) None of these

20. In a 2 dimensional differential dynamical system, if community matrix at equilibrium point has one positive and one negative eigen values then equilibrium point is called:

(A) saddle

(B) attracting spiral

(C) repelling spiral

(D) None of these

Section - B

(Very Short Answer Type Questions)

(1½ marks each)

Note: Answer in 2-3 sentences.

1. What is logistic grow population model? Explain interinsic growth rate and carraying capacity.
2. Write limitations of exponetial growth model.
3. Explain linear and non-linear discrete population models with examples.
4. Find the solution of the difference equation $x_{n+1} = \frac{1}{3} x_n$, $x_0 = 2$
5. Explain graphical repesesentation of solving difference equation.
6. Write Kolmogrov model and explain its parameter.

7. State diffenent categories (types) of two interacting populations.
8. Define community matix and explain its role in stability analysis.
9. Differentiate between constant-yield and constant effort harvesting models.
10. Write lotkka-voltera differential equation model and explain its parameters.

Section - C

(Short Answer Type Questions)

(2½ marks each)

Note: Attempt all questions.

1. Derive exponential growth differential equation model.
2. Explain the role of parameter r in stability of exponential growth model $\frac{dx}{dt} = r(x),\ x(0) = x_0 > 0$.
3. Fomd equilibria and its stability of logistic difference equation $x_{n+1} = x_n + rx_n\left(1 - \frac{x_n}{K}\right)$.
4. Find equilibria of difference equation $x_{n+1} = \frac{2x_n}{1+X_n}$ and determine whether it is asymptotically stable or unstable.

5. Describe the orbits of the system $x^1 = e^{-y}$, $y^1 = e^x$.
6. Linearize the system at each equilibrium.

$$x^1 = y+1,\ y^1 = x^2 + y$$

7. What is periodic orbit? Explain periodic orbit in a competetion model.
8. Discuss local asymptotic stablity of a two-dimential differential system.
9. Find all equilibria of following mutualistic system and determine its stability.

$$x' = x(-20 - x + 2y),\ y' = y(-50 + x - y),$$

10. For the difference equation $x_n + 1 = x_n^2 - 4x_n + 6$; find all equilibria and determine its stability.

Section - D

(Long Answer Type Questions)

(4 marks each)

Note- Attempt all questions.

1. Show that the solution of logistic population model

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right),\ x(0) = x_0 \text{ is } x(t) = \frac{Kx_0}{x_0 + (K - x_0)e^{-rt}}$$

Or

Find all equilibria and determine which are asymptotically stable for

$$x^1 = rx\log\left(\frac{K}{x}\right)$$

2. Find general solution of second order difference equation

$$x_{n+2} - 3x_{n+1} + 2x_n = 0$$

Or

Find period 2 orbit (all points of period 2) for the difference equation

$$x_{n+1} = x_n + rx_n(1 - x_n)$$

For which value of r periodic orbit is stable?

3. For a cell differentiation model

$$\frac{dx}{dt} = y - x,\ \frac{dy}{dt} = \frac{5x^2}{4tx^2} - y$$

determine the equilibrium points.

Linearize the system at each equilibrium point and determine the local stability of positive equilibrium points.

Or

Consider the system $x^1 = y$, $y^1 = -x - x^3$. Show that (0,0) is the only equilibrium and also show that the function $V(x,y) = x^2 + y^2$ decreases along every orbit and tends to zero, showing that (0,0) is asymptotically stable.

4. Determine the outcome of a competition modeled by the system

$$x^1 = x\,(100 - 4x - y),\ y^1 = y\,(60 - x - 2y)$$

Or

Determine the qualitative behaviour of a prey-predator modeled by

$$x^1 = x\left(1-\frac{x}{30}\right)-\frac{xy}{x+10},\ \ y^1 = y\left(\frac{x}{x+10}-\frac{1}{3}\right)$$

5. Find all equilibria of the following mutualistic system and determine its stablity:

$$x^1 = x(-20-x+2y);\ \ y^1 = y(-50+x-y)$$

Or

Determine the response of the system

$$x^1 = x(100-4x-y),\ \ y^1 = y(60-x-2y)$$

to constant-effort harvesting of the x-species.